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PREGEOMETRICAL FORMULATION
OF
BERKOVITS' OPEN RNS
SUPERSTRING FIELD THEORIES

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Abstract

We propose a pregeometrical formulation of Berkovits' open Ramond-Neveu-Schwarz(RNS) superstring field theories. We show that Berkovits' open RNS superstring field theories arise by expanding around particular solutions of the classical equations of motion for this theory. Our action contains pure ghost operators only and so is formally background independent.

1 Introduction

Tachyon condensation has attracted great interests after the proposal given by Sen [1]. It is known that the number of space-time supersymmetry and the space-time geometry changes during tachyon condensation. A classical analysis of this process was given in [2] assuming Sen's conjecture. In order to investigate this process without this assumption, one needs an off-shell formulation of superstring theory of the Gliozzi-Scherk-Olive (GSO) $(-)$ sector as well as the GSO $(+)$ sector, because the vacuum structure changes during tachyon condensation. If one would like to extract some information on space-time supersymmetry, the Ramond sector must be included in the study. For this purpose, GSO unprojected superstring field theories of a Neveu-Schwarz (NS) string field and a Ramond (R) string field is expected to be powerful.

Right after the construction of open bosonic string field theory (SFT) [3] of a fermionic string field V of -1 ghost number, Witten extended it to open superstring field theory (super-SFT) [4] of an NS string field V_{NS} of -1 picture and a R string field V_R of $-\frac{1}{2}$ picture. The NS interaction term of the theory must contain the picture changing operator Z of $+1$ picture because the non-vanishing small Hilbert space norm $\langle c\partial c\partial^2 c e^{-2\phi} \rangle = 1$ carries -2 picture. It was shown [5] that the collision of two Z 's causes the contact term divergences which prevent the action from being gauge-invariant. Berkovits gave a solution [6, 7, 8] to this contact term problem, which was to use a bosonic NS string field Φ of zero picture and to construct the open NS super-SFT action to be a Wess-Zumino-Witten like form in the large Hilbert space. It is not possible to construct an action for a single R string field Ψ of zero ghost number and $\frac{1}{2}$ picture. The kinetic term of such a R string field must contain the inverse picture changing operator Y of -1 picture in the large Hilbert space. The inverse picture changing operator Y causes inconsistencies due to the non-trivial kernel. Berkovits provided [6] the $N=1$ $d=4$ super-Poincare invariant open super-SFT action including R string fields as well as an NS string field, where he used an alternative method which is to split a R string field Ψ into two string fields, Ψ and $\bar{\Psi}$, with $\frac{1}{2}$ and $-\frac{1}{2}$ picture respectively. However it was not clear how to extend this construction to the other dimensions. Recently in [9], Berkovits beautifully extended this construction to the other dimensions and provided actions of open RNS super-SFT's. In this paper, we work with this theory.

It was shown [10] that Witten's bosonic SFT can be obtained from a pure cubic SFT which does not contain a Becchi-Rouet-Stora-Tyupin (BRST) operator and so is formally background independent. Thus pure cubic SFT can be viewed as a fundamental formulation of Witten's bosonic SFT. Recently, Berkovits' NS super-SFT was shown to be formulated pregeometrically by Klusoň in [11]. The pregeometrical action contains a pure ghost operator Q_0 which is related to the BRST operator Q by a similarity transformation and has non-trivial cohomology. So this theory describes physical open string excitations.

In this paper, we address a question whether Berkovits' RNS super-SFT's can be formulated pregeometrically. The pregeometrical super-SFT will contain a pure ghost operator Q_0 as in [11]. Berkovits' d=8,4 Lorentz covariant open RNS super-SFT's contain four operators while we have only two background independent operators Q_0 and the zero mode of η . So we must 'create' two operators as was done in the pure cubic SFT where the BRST operator was absent at first and derived by expanding around solutions of the equations of motion. This implies that the pregeometrical action will take a different form from the actions of Berkovits' d=8,4 Lorentz covariant open RNS super-SFT's while the Berkovits' NS super-SFT action and the pregeometrical action had the same form [11].

The plan of the present paper is as follows. In the section 2, some relevant aspects of Berkovits' open RNS super-SFT's are reviewed. In the section 3, after reviewing the pregeometrical formulation of Witten's open bosonic SFT, we propose a pregeometrical action for manifest d=8,4 Lorentz covariant open RNS super-SFT's reviewed in the section 2. The last section is devoted to a summary and conclusions.

2 Review of Berkovits' Open RNS Superstring Field Theories

In this section we review some relevant aspects of open RNS super-SFT's constructed by Berkovits in [9].

The RNS superstring theory is constructed from a combined system of the $c=15$ superconformal 'matter' system and the $c=-15$ superconformal ghost system of a set of fermionic ghosts (b, c) and a set of bosonic ghosts (β, γ) . It is convenient to feminize the bosonic ghosts (β, γ) as $\beta = \partial\xi e^{-\phi}$ and

$\gamma = \eta e^\phi$. The physical states are defined by non-trivial elements of the BRST cohomology in the small Hilbert space. The BRST operator Q is defined [12] by¹

$$Q = \oint j, \quad j = e^{-S}(-be^{2\phi}\eta\partial\eta)e^S, \quad S = \oint(cG_me^{-\phi}\xi + \frac{1}{4}\partial e^{-2\phi}\xi\partial\xi c\partial c). \quad (2.1)$$

This BRST operator is equivalent to the ordinary BRST operator but the BRST current j is different by a total derivative term and the OPE of two BRST currents is non-singular, $j(z)j(w) \sim 0$.

As was explained in the introduction, we use three string fields $[\Phi, \Psi, \bar{\Psi}]$ of $[0, \frac{1}{2}, -\frac{1}{2}]$ picture and zero ghost number.² Berkovits proposed actions for open RNS super-SFT's supposing that one can define a conserved C-charge as

$$\begin{array}{c|ccccccc} & \Phi & \Psi & \bar{\Psi} & (b, c) & (\beta, \gamma) & (\xi, \eta) & e^{n\phi} \\ \hline \text{C-charge} & 0 & -\frac{1}{3} & \frac{1}{3} & (0, 0) & (0, 0) & (1, -1) & n \end{array}. \quad (2.2)$$

The non-vanishing large Hilbert space norm $\langle \xi c \partial c \partial^2 c e^{-2\phi} \rangle = 1$ carries -1 C-charge. Assuming that Q does not carry C-charge, the simplest action was constructed as

$$\begin{aligned} S = & \left\langle \frac{1}{2}(g^{-1}\tilde{\eta}g)(g^{-1}Qg) + \frac{1}{2}\int_0^1 dt(\hat{g}^{-1}\partial\hat{g})\{\hat{g}^{-1}\tilde{\eta}\hat{g}, \hat{g}^{-1}Q\hat{g}\} \right. \\ & \left. + g^{-1}(Q\bar{\Psi})g(\tilde{\eta}\Psi) - \frac{1}{3}\bar{\Psi}(Q\bar{\Psi})^2 + \frac{1}{3}\Psi(\tilde{\eta}\Psi)^2 \right\rangle, \quad (2.3) \\ \text{with } & g = e^\Phi, \quad \hat{g} = e^{\hat{\Phi}(t)}, \quad \hat{\Phi}(0) = 0, \quad \hat{\Phi}(1) = \Phi, \end{aligned}$$

where $\tilde{\eta}$ stands for the zero-mode of η and we defined $\{, \}$ by $\{X, Y\} = X \star Y - (-1)^{X \cdot Y} Y \star X$ with $(-1)^{X \cdot Y} = -1$ only for both X and Y are Grassmann odd. The nonlinear equations of motion obtained varying this action can be realized as $[-\frac{2}{3}, -1, -\frac{4}{3}]$ C-charge part of the equation

$$(G + A)^2 = 0, \quad (2.4)$$

$$G = G_0 + G_{-1}, \quad G_0 = Q, \quad G_{-1} = \tilde{\eta},$$

$$A = A_0 + A_{-\frac{1}{3}} + A_{-\frac{2}{3}}, \quad A_0 = g^{-1}(Qg), \quad A_{-\frac{1}{3}} = g^{-1}(Q\bar{\Psi})g, \quad A_{-\frac{2}{3}} = \tilde{\eta}\Psi,$$

¹We use a symbol \oint as $\oint \frac{dz}{2\pi i}$ throughout this paper.

²The picture number operator P and the ghost number operator J_g are defined by $P = \oint(\xi\eta - \partial\phi)$ and $J_g = \oint(\eta\xi + cb)$.

where G_n and A_n carry n C-charge and the rest part of $[0, -\frac{1}{3}, -\frac{5}{3}]$ C-charge is automatically satisfied. We can define a ‘chiral’ (‘anti-chiral’) field Ω ($\bar{\Omega}$) by $\tilde{\eta}\Omega = 0$ ($Q\bar{\Omega} = 0$). Since the cohomologies of Q and $\tilde{\eta}$ are trivial in the large Hilbert space, Ω ($\bar{\Omega}$) can be written as $\tilde{\eta}\Psi$ ($Q\bar{\Psi}$) for some Ψ ($\bar{\Psi}$). Therefore one can treat Ω and $\bar{\Omega}$ as fundamental string fields. In terms of Ω and $\bar{\Omega}$, the second line of the action (2.3) can be expressed as, $\langle g^{-1}\bar{\Omega}g\Omega \rangle - \frac{1}{3}\langle \bar{\Omega}^3 \rangle_{\bar{F}} + \frac{1}{3}\langle \Omega^3 \rangle_F$, where we defined the small Hilbert space norms $\langle \tilde{\eta}(\xi e^{-2\phi} c \partial c \partial^2 c) \rangle_F = 1$ and $\langle Q(\xi e^{-2\phi} c \partial c \partial^2 c) \rangle_{\bar{F}} = 1$.

The C-charge assignment (2.2) implies that the world-sheet ‘matter’ fields will carry non-zero C-charge. This causes the BRST operator Q to carry C-charge in general because the ‘matter’ superconformal generators T_m and G_m appear in Q . There are two ways to respect the C-charge assignment (2.2). First, we generalize Q to carry C-charge below, which enables us to construct manifestly d=8,4 Lorentz covariant open RNS super-SFT’s. Secondly, as will be done in the next section, we use the action (2.3) with the pure ghost operator Q_0 , instead of Q , as a pregeometrical formulation.

Now we generalize $G = Q + \tilde{\eta}$ to $G = G_0 + G_{-\frac{1}{3}} + G_{-\frac{2}{3}} + G_{-1}$, which implies that Q can carry non-zero C-charge. One can show that $(G+A)^2 = 0$ implies consistent equations of motion when

$$\begin{aligned} A &= A_0 + A_{-\frac{1}{3}} + A_{-\frac{2}{3}}, \\ A_0 &= g^{-1}(G_0 g), \quad A_{-\frac{1}{3}} = g^{-1}(G_{-\frac{1}{3}} g) + g^{-1}\bar{\Omega}g, \quad A_{-\frac{2}{3}} = \Omega, \end{aligned} \quad (2.5)$$

where we defined a chiral (anti-chiral) string fields Ω ($\bar{\Omega}$) by $G_{-1}\Omega = 0$ ($G_0\bar{\Omega} = 0$) which implies that $\Omega = G_{-1}\Psi$ ($\bar{\Omega} = G_0\bar{\Psi}$) for some Ψ ($\bar{\Psi}$), assuming the triviality of the cohomologies of G_0 and G_{-1} . The equations of motion can be obtained varying the action

$$\begin{aligned} S &= \left\langle \frac{1}{2}(g^{-1}G_{-1}g)(g^{-1}G_0g) + \frac{1}{2}(g^{-1}G_{-\frac{2}{3}}g)(g^{-1}G_{-\frac{1}{3}}g) \right. \\ &\quad \left. + \frac{1}{2} \int_0^1 dt (\hat{g}^{-1} \partial \hat{g}) \left(\{ \hat{g}^{-1} G_{-1} \hat{g}, \hat{g}^{-1} G_0 \hat{g} \} + \{ \hat{g}^{-1} G_{-\frac{2}{3}} \hat{g}, \hat{g}^{-1} G_{-\frac{1}{3}} \hat{g} \} \right) \right. \\ &\quad \left. + g^{-1}\bar{\Omega}g\Omega + \bar{\Omega}g(G_{-\frac{2}{3}}g^{-1}) - \Omega g^{-1}(G_{-\frac{1}{3}}g) \right\rangle \\ &\quad - \left\langle \frac{1}{2}\bar{\Omega}G_{-\frac{1}{3}}\bar{\Omega} + \frac{1}{3}(\bar{\Omega})^3 \right\rangle_{\bar{F}} + \left\langle \frac{1}{2}\Omega G_{-\frac{2}{3}}\Omega + \frac{1}{3}(\Omega)^3 \right\rangle_F \end{aligned} \quad (2.6)$$

where $\langle \rangle_{\bar{F}}$ and $\langle \rangle_F$ are defined using the small Hilbert space norms $\langle G_{-1}(\xi e^{-2\phi} c \partial c \partial^2 c) \rangle_F = 1$ and $\langle G_0(\xi e^{-2\phi} c \partial c \partial^2 c) \rangle_{\bar{F}} = 1$.

Different choice of the definition of C-charge leads to different background geometry. Let us define C-charge as

$$C = P + \frac{1}{3} \oint j_N, \quad j_N = \begin{cases} \psi^0 \psi^9, & \text{for } d=8, \\ i(\psi^4 \psi^5 + \psi^6 \psi^7 + \psi^8 \psi^9), & \text{for } d=4, \end{cases} \quad (2.7)$$

where P is the picture, so that world-sheet ‘matter’ fields $\psi^\pm = \frac{1}{\sqrt{2}}(\psi^0 \pm \psi^9)$ for $d=8$ and $\psi^{\pm j} = \frac{1}{\sqrt{2}}(\psi^{2j+2} \pm i\psi^{2j+3})$, $j = 1, 2, 3$, for $d=4$ carry $\pm \frac{1}{3}$ C-charge.³ Since j_N for $d=4$ is defined to be anti-hermitian, the string fields and the operators must satisfy the hermiticity condition, $\Phi^\dagger = -\Phi$, $\Psi^\dagger = \bar{\Psi}$, $G_0^\dagger = G_{-1}$, $G_{-\frac{1}{3}}^\dagger = G_{\frac{2}{3}}$, while, for $d=8$, they are independent each other. For $d=4$, j_N can be generalized to $j_N = \partial H$ which is the $U(1)$ current on a Calabi-Yau manifold.

The operator G , which carries $[0, -\frac{1}{3}, -\frac{2}{3}, -1]$ C-charge, was defined by a similarity transformation of $Q + \tilde{\eta}$ as

$$\begin{aligned} G &= e^R (Q + \tilde{\eta}) e^{-R} \quad \text{with } R = \oint c \xi e^{-\phi} \psi^{+j} \partial x^{-j}, \\ &= e^{-U} Q_0 e^U + \oint \eta e^{\phi} \psi^{-j} \partial x^{+j} + \oint c e^{-\phi} \psi^{+j} \partial x^{-j} + \tilde{\eta}, \end{aligned} \quad (2.8)$$

where $(\psi^{\pm j}, x^{\pm j})$ stands for (ψ^\pm, x^\pm) for $d=8$, and U is

$$U = \oint \left(c \xi e^{-\phi} \psi^p \partial x_p + \frac{1}{2} (\partial \phi + j_N) c \partial c \xi \partial \xi e^{-2\phi} \right) \quad (2.9)$$

where p runs from 0 to 3 for $d=4$ and from 1 to 8 for $d=8$. One can show that the operators G_0 and G_{-1} are nilpotent and have trivial cohomologies in the large Hilbert space. The action (2.6) with G_n obtained above provides manifest $d=8, 4$ Lorentz covariant open RNS super-SFT’s.⁴

In order to relate to the manifestly $N=1$ $d=4$ super-Poincare covariant open super-SFT action, Berkovits performed a similarity transformation further as $\hat{G} \equiv e^{\frac{1}{2}U} G e^{-\frac{1}{2}U}$ and showed that \hat{G} can be rewritten in an $N=1$ $d=4$ super-Poincare covariant notation using Green-Schwarz-like variables [13].

³Similarly we define $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^9)$ for $d=8$ and $x^{\pm j} = \frac{1}{\sqrt{2}}(x^{2j+2} \pm i x^{2j+3})$, $j = 1, 2, 3$, for $d=4$.

⁴One can define similarly a $d=0$ open RNS super-SFT.

3 Pregeometrical Formulation

In this section, we will show that Berkovits' RNS super-SFT reviewed in the previous section can be formulated pregeometrically. To do this we begin with reviewing the simplest example of such a pregeometrical formulation for Witten's bosonic SFT.

3.1 Pure Cubic String Field Theory

In [10], a pregeometrical SFT action for Witten's open bosonic SFT was proposed as a pure cubic form

$$S = \left\langle \frac{1}{3} V \star V \star V \right\rangle. \quad (3.1)$$

The equation of motion is $V \star V = 0$. Expanding around a classical solution $V = V_0 + v$, we obtain an action of the fluctuation v

$$S = \left\langle \frac{1}{2} v \star D_{V_0} v + \frac{1}{3} v \star v \star v \right\rangle \quad (3.2)$$

where D_{V_0} is defined by $D_{V_0} X = V_0 \star X - (-1)^X X \star V_0$ and is a derivation. If we can find a classical solution V_0 such that $D_{V_0} = Q$, where Q is the BRST operator of bosonic string theory, then the action turns out to be the Witten's open bosonic SFT action. In order to achieve this, we introduce the half-string formulation [4]

$$\mathcal{I} \star X = X \star \mathcal{I} = X, \quad \forall X, \quad (3.3)$$

$$Q^R \mathcal{I} = -Q^L \mathcal{I}, \quad (3.4)$$

$$Q^R X \star Y = -(-1)^X X \star Q^L Y \quad \forall X, Y, \quad (3.5)$$

where $Q^{L,R}$ is the BRST operator integrated over the left/right half of the string ($Q = Q^L + Q^R$) and \mathcal{I} is the identity operator of the algebra. It was shown that

$$V_0 = Q^L \mathcal{I} \quad (3.6)$$

obeys the equation of motion and satisfies, $D_{Q^L \mathcal{I}} X = QX$, for any string field X , using (3.3)-(3.5) and the fact

$$\{Q, j\} = 0, \quad Q = \oint j, \quad j = cT_m + bc\partial c + \frac{3}{2}\partial^2 c. \quad (3.7)$$

As a result, Witten's SFT has arisen through a classical solution of pure cubic SFT.

3.2 Pregeometrical Formulation of Berkovits' Open RNS Superstring Filed Theories

In [11], Berkovits' open NS super-SFT was formulated pregeometrically, where the pregeometrical action includes two operators, Q_0 and $\tilde{\eta}$, and has the same form as the original open NS super-SFT action. In this section, we will provide a pregeometrical formulation of manifest d=8,4 Lorentz covariant RNS super-SFT's reviewed in the section 2.

As was mentioned in the section 2, the C-charge assignment (2.2) make the world-sheet 'matter' fields to carry non-zero C-charge. This causes the BRST charge Q to carry C-charge in general. Thus we use the action (2.3) with the pure ghost operator, $Q_0 = -\oint b e^{2\phi} \eta \partial \eta$, instead of Q . This action is formally background independent and provides a pregeometrical formulation of open RNS super-SFT as will be shown bellow.

We propose an action of pregeometrical open RNS super-SFT

$$\begin{aligned}
S = & \left\langle \frac{1}{2} (g^{-1} \tilde{\eta} g) (g^{-1} Q_0 g) + \frac{1}{2} \int_0^1 dt (\hat{g}^{-1} \partial_t \hat{g}) \{ \hat{g}^{-1} \tilde{\eta} g, \hat{g}^{-1} Q_0 g \} \right. \\
& \left. + g^{-1} (Q_0 \bar{\Psi}) g (\tilde{\eta} \Psi) - \frac{1}{3} \bar{\Psi} (Q_0 \bar{\Psi})^2 + \frac{1}{3} \Psi (\tilde{\eta} \Psi)^2 \right\rangle \quad (3.8)
\end{aligned}$$

with $Q_0 = - \int \eta \partial \eta e^{2\phi} b$, $g = e^\Phi$, $\hat{g} = e^{\hat{\Phi}(t)}$, $\hat{\Phi}(0) = 0$, $\hat{\Phi}(1) = \Phi$,

where the operator Q_0 is a pure ghost operator which is related to the BRST operator Q by a similarity transformation (2.1).

As before, $(G + A)^2 = 0$ implies the equations of motion when

$$\begin{aligned}
G &= G_0 + G_{-1}, & A &= A_0 + A_{-\frac{1}{3}} + A_{-\frac{2}{3}}, \\
G_0 &= Q_0, & A_0 &= g^{-1} (Q_0 g), \\
G_{-1} &= \tilde{\eta}, & A_{-\frac{1}{3}} &= g^{-1} (Q_0 \bar{\Psi}) g, \\
& & A_{-\frac{2}{3}} &= \tilde{\eta} \Psi.
\end{aligned} \quad (3.9)$$

The operator G is nilpotent because $Q_0^2 = 0$, $\tilde{\eta}^2 = 0$ and $\{Q_0, \tilde{\eta}\} = 0$. The equation $(G + A)^2 = 0$ is invariant under the gauge transformation,

$\delta A = G\sigma + [A, \sigma]$, where

$$\begin{aligned}\sigma &= \sigma_{\frac{1}{3}} + \sigma_0 + \sigma_{-\frac{1}{3}}, \\ \sigma_{\frac{1}{3}} &= \{Q_0 + A_0, \Lambda_{\frac{1}{3}}\}, \quad \sigma_0 = \tilde{\eta}\Lambda_1 + \{A_{-\frac{2}{3}}, \Lambda_{\frac{2}{3}}\}, \quad \sigma_{-\frac{1}{3}} = \tilde{\eta}\Lambda_{\frac{2}{3}}.\end{aligned}\quad (3.10)$$

Now, let us examine an expansion around a classical solution Φ_0 , Ψ_0 and $\bar{\Psi}_0$

$$g = g_0 h, \quad g_0 = e^{\Phi_0}, \quad h = e^\phi, \quad \Psi = \Psi_0 + \psi, \quad \bar{\Psi} = \bar{\Psi}_0 + \bar{\psi}. \quad (3.11)$$

Substituting (3.11) into A of (3.9), we find that A is expanded as, $A = \bar{A} + a$, where

$$\begin{aligned}\bar{A}_0 &= g_0^{-1}(Q_0 g_0), & a_0 &= h^{-1}(Qh + [g_0^{-1}(Qg_0), h]), \\ \bar{A}_{-\frac{1}{3}} &= g_0^{-1}(Q_0 \bar{\Psi}_0)g_0, & a_{-\frac{1}{3}} &= h^{-1}[g_0^{-1}\bar{\Omega}_0 g_0, h] + h^{-1}g_0^{-1}Q\bar{\psi}g_0 h, \\ \bar{A}_{-\frac{2}{3}} &= \tilde{\eta}\Psi_0, & a_{-\frac{2}{3}} &= \tilde{\eta}\psi.\end{aligned}\quad (3.12)$$

These a 's satisfy the equation $(\tilde{G} + a)^2 = 0$ with

$$\tilde{G}X = GX + \{\bar{A}, X\}. \quad (3.13)$$

The nilpotency of the operator \tilde{G} follows immediately from the nilpotency $G^2 = 0$, the equation of motion $G\bar{A} + \bar{A}^2 = 0$ and a Bianchi identity $\{\bar{A}, \{\bar{A}, X\}\} + \{X, \bar{A}^2\} = 0$. The definition (3.13) implies that we can define operators \tilde{G}_n by

$$\begin{aligned}\tilde{G}_0 X &= G_0 X + \{\bar{A}_0, X\}, \\ \tilde{G}_{-\frac{1}{3}} X &= \{\bar{A}_{-\frac{1}{3}}, X\}, \\ \tilde{G}_{-\frac{2}{3}} X &= \{\bar{A}_{-\frac{2}{3}}, X\}, \\ \tilde{G}_{-1} X &= G_{-1} X.\end{aligned}\quad (3.14)$$

It is straightforward to see that the nilpotency of \tilde{G} is expressed as a set of equations satisfied by the operators \tilde{G}_n order by order of C-charge;

$$\begin{aligned}\tilde{G}_0^2 &= 0, \\ \{\tilde{G}_0, \tilde{G}_{-\frac{1}{3}}\} &= 0, \\ \{\tilde{G}_0, \tilde{G}_{-\frac{2}{3}}\} + \tilde{G}_{-\frac{1}{3}}^2 &= 0,\end{aligned}$$

$$\begin{aligned}
\{\tilde{G}_0, \tilde{G}_{-1}\} + \{\tilde{G}_{-\frac{1}{3}}, \tilde{G}_{-\frac{2}{3}}\} &= 0, \\
\{\tilde{G}_0, \tilde{G}_{-1}\} + \tilde{G}_{-\frac{2}{3}}^2 &= 0, \\
\{\tilde{G}_{-\frac{2}{3}}, \tilde{G}_{-1}\} &= 0, \\
\tilde{G}_{-1}^2 &= 0,
\end{aligned} \tag{3.15}$$

which imply that the operators \tilde{G}_0 and \tilde{G}_{-1} are nilpotent. Using these \tilde{G}_n , we find that a 's in (3.12) can be re-expressed compactly as

$$\begin{aligned}
a_0 &= h^{-1}(\tilde{G}_0 h), \\
a_{-\frac{1}{3}} &= h^{-1}(\tilde{G}_{-\frac{1}{3}} h) + h^{-1} \bar{\omega} h, \\
a_{-\frac{2}{3}} &= \omega,
\end{aligned} \tag{3.16}$$

where we defined ‘chiral’ and ‘anti-chiral’ string fields by $\omega = \tilde{G}_{-1} \psi$ and $\bar{\omega} = \tilde{G}_0(g_0^{-1} \bar{\psi} g_0)$. Note that a 's of (3.16) have the same structure as A of (2.5) in the section 2. So we can conclude that the fluctuation string fields ϕ , ω and $\bar{\omega}$ satisfy the same equations of motion as those for open RNS super-SFT's of the action (2.6). This implies that our super-SFT of the action (3.8) reproduces open RNS super-SFT's of the actions (2.6), if we can find classical solutions, Φ_0 , Ω_0 and $\bar{\Omega}_0$, which reproduce the corresponding operator \tilde{G} .

In the rest of this section, we concentrate on solving the equation (3.13) in terms of Φ_0 , Ω_0 and $\bar{\Omega}_0$ with G being

$$G = Q_0 + \tilde{\eta}, \tag{3.17}$$

and \tilde{G} being G in the equation (2.8)

$$\begin{aligned}
\tilde{G} &= e^R(Q + \tilde{\eta})e^{-R} \\
&= e^{-U} Q_0 e^U + \oint \eta e^{\phi} \psi^{-j} \partial x^{+j} + \oint c e^{-\phi} \psi^{+j} \partial x^{-j} + \tilde{\eta},
\end{aligned} \tag{3.18}$$

where $(\psi^{\pm j}, x^{\pm j}) = (\psi^{\pm}, x^{\pm})$ for $d=8$ and j runs 1 to 3 for $d=4$. U is defined in the equation (2.9).

For this purpose, we introduce the half-string formulation [4] as before,

$$(A) \quad \mathcal{I} \star X = X \star \mathcal{I} = X, \quad \forall X, \tag{3.19}$$

$$(B) \quad G^R \mathcal{I} = -G^L \mathcal{I}, \tag{3.20}$$

$$(C) \quad G^R(X) \star Y + (-1)^X X \star G^L(Y) = 0, \quad \forall X, Y, \tag{3.21}$$

where \mathcal{I} is the identity operator of the algebra. Using the above definition, one can show [11] that

$$\bar{A} = (\tilde{G} - G)^L \mathcal{I} \quad (3.22)$$

satisfies the equation (3.13) because

$$\begin{aligned} \{\bar{A}, X\} &= (\tilde{G} - G)^L \mathcal{I} \star X - (-1)^X X \star (\tilde{G} - G)^L \mathcal{I} \\ &\stackrel{(B)}{=} -(\tilde{G} - G)^R \mathcal{I} \star X - (-1)^X X \star (\tilde{G} - G)^L \mathcal{I} \\ &\stackrel{(C)}{=} \mathcal{I} \star (\tilde{G} - G)^L X + (\tilde{G} - G)^R X \star \mathcal{I} \\ &\stackrel{(A)}{=} (\tilde{G} - G) X \end{aligned} \quad (3.23)$$

and the equation $(G + \bar{A})^2 = 0$ because

$$\begin{aligned} G\bar{A} &= (G^L + G^R)(\tilde{G} - G)^L \mathcal{I} = G^L \tilde{G}^L \mathcal{I} + G^R \tilde{G}^L \mathcal{I} \\ &\stackrel{(C)}{=} G^L \tilde{G}^L \mathcal{I} + \tilde{G}^L \mathcal{I} \star G^L \mathcal{I} \stackrel{(B)}{=} G^L \tilde{G}^L \mathcal{I} - \tilde{G}^R \mathcal{I} \star G^L \mathcal{I} \\ &\stackrel{(C)(A)}{=} G^L \tilde{G}^L \mathcal{I} + \tilde{G}^L G^L \mathcal{I} = \{\tilde{G}^L, G^L\} \mathcal{I}, \end{aligned} \quad (3.24)$$

$$\begin{aligned} \bar{A}^2 &= (\tilde{G} - G)^L \mathcal{I} \star (\tilde{G} - G)^L \mathcal{I} \stackrel{(B)}{=} -(\tilde{G} - G)^R \mathcal{I} \star (\tilde{G} - G)^L \mathcal{I} \\ &\stackrel{(C)(A)}{=} (\tilde{G} - G)^L (\tilde{G} - G)^L \mathcal{I} = -\{\tilde{G}^L, G^L\} \mathcal{I} \end{aligned} \quad (3.25)$$

where we indicated which equation was used on an equal sign. In the calculation, we used equations $(G^L)^2 = G^L G^R = (\tilde{G}^L)^2 = 0$ which follow from the facts that the OPE of the two currents $j(z)$ of $G = \oint j(z)$ is non-singular

$$j(z)j(w) \sim 0, \quad j(z) = -\eta \partial \eta e^{2\phi} b + \eta, \quad (3.26)$$

and that the OPE of the two currents $\tilde{j}(z)$ of $\tilde{G} = \oint \tilde{j}$ is non-singular

$$\tilde{j}(z)\tilde{j}(w) \sim 0, \quad \tilde{j} = e^R(j_{BRST} + \eta)e^{-R}. \quad (3.27)$$

Thus we have shown that the equation (3.22) solves the equation (3.13).

Now we solve the equation (3.22) in terms of Φ_0 , Ω_0 and $\bar{\Omega}_0$. The definition (3.12) of \bar{A} implies that the equation (3.22) can be written, order by order of C-charge, as

$$g_0^{-1}(Q_0 g_0) = (\tilde{G}_0^L - Q_0^L) \mathcal{I}, \quad (3.28)$$

$$g_0^{-1} \bar{\Omega}_0 g_0 = \tilde{G}_{-\frac{1}{3}}^L \mathcal{I}, \quad (3.29)$$

$$\Omega_0 = \tilde{G}_{-\frac{2}{3}}^L \mathcal{I}, \quad (3.30)$$

$$0 = (\tilde{G}_{-1} - \tilde{\eta})^L \mathcal{I}. \quad (3.31)$$

The first equation (3.28) is satisfied by

$$\Phi_0 = U^L \mathcal{I}, \quad (3.32)$$

because one can show [11] that

$$\begin{aligned} e^{-U^L \mathcal{I}} \star Q_0 e^{U^L \mathcal{I}} &= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!} \overbrace{[U^L \mathcal{I}, \dots, [U^L \mathcal{I}, Q_0(U^L \mathcal{I})] \dots]}^{n-1} \\ &= \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n!} \overbrace{[U^L, \dots, [U^L, Q_0^L] \dots]}^n \mathcal{I} \\ &= (e^{-U} \star Q_0 e^U - Q_0)^L \mathcal{I}. \end{aligned} \quad (3.33)$$

In the second equality in (3.33), we used a relation

$$\{A^R, B^L\} = 0, \quad \forall A, B, \quad (3.34)$$

which follows from the fact that one can show that, using the algebra (3.19)-(3.21), for any string field X ,

$$\begin{aligned} A^R B^L X &\stackrel{(A)(C)}{=} -(-1)^{A \cdot BX} B^L X \star A^L \mathcal{I} \stackrel{(A)(C)}{=} (-1)^{A \cdot BX} B^R \mathcal{I} \star X \star A^L \mathcal{I} \\ &\stackrel{(C)(A)}{=} -(-1)^{A \cdot BX + A \cdot X} B^R \mathcal{I} \star A^R X \stackrel{(C)(A)}{=} (-1)^{A \cdot BX + A \cdot X} B^L A^R X \\ &= (-1)^{A \cdot B} B^L A^R X. \end{aligned} \quad (3.35)$$

In the third equality in (3.33), we used a relation, $\{A, B\}^L = \{A^L, B^L\}$, which follows from the equation (3.34).

The second equation (3.29) is solved by

$$\bar{\Omega}_0 = e^{U^L} \alpha^L e^{-U^L} \mathcal{I}, \quad \alpha^L = \oint_L \eta e^\phi \psi^{-j} \partial x^{+j}, \quad (3.36)$$

because it follows that

$$\begin{aligned} e^{U^L \mathcal{I}} \star \alpha^L \mathcal{I} \star e^{-U^L \mathcal{I}} &= \sum_{n=0}^{\infty} \frac{1}{n!} \overbrace{[U^L \mathcal{I}, \dots, [U^L \mathcal{I}, \alpha^L \mathcal{I}] \dots]}^n \\ &= \sum_{n=0}^{\infty} \frac{1}{n!} \overbrace{[U^L, \dots, [U^L, \alpha^L] \dots]}^n \mathcal{I} \\ &= e^{U^L} \alpha^L e^{-U^L} \mathcal{I}. \end{aligned} \quad (3.37)$$

The third equation (3.30) implies that

$$\Omega_0 = \beta^L \mathcal{I}, \quad \beta^L = \oint_L c e^{-\phi} \psi^{+j} \partial x^{-j}. \quad (3.38)$$

The last equation (3.31) is automatically satisfied because $\tilde{G}_{-1} = \tilde{\eta}$.

As a result, manifest d=8,4 Lorentz covariant open RNS super-SFT's are shown to be obtained from the pregeometrical super-SFT (3.8) using a set of classical solutions (3.32), (3.36) and (3.38). The uniqueness of the classical solution for the particular \tilde{G} can be shown as in [10].

4 Summary

In this paper, we proposed a pregeometrical formulation for Berkovits' manifest d=8,4 Lorentz covariant open RNS super-SFT's. The pregeometrical action contains a pure ghost operator Q_0 instead of the BRST operator Q . We showed that manifest d=8,4 Lorentz covariant open RNS super-SFT's are derived from the pregeometrical super-SFT by expanding around solutions of the equations of motion. The pregeometrical action takes a different form from the manifest d=8,4 Lorentz covariant open RNS super-SFT actions as in the pure cubic SFT [10], while the Berkovits' NS super-SFT action and the pregeometrical action had the same form [11].

We derived open RNS super-SFT's from the pregeometrical super-SFT at the level of their equations of motion. We expect that this can be done at the level of actions also.

We used the operator G of (2.8) which leads to manifest d=4 Lorentz covariant RNS super-SFT, while Berkovits showed that N=1 d=4 super-Poincare covariant RNS super-SFT can be obtained using a similarity transform $\hat{G} = e^{\frac{1}{2}U} G e^{-\frac{1}{2}U}$ in [9]. It is interesting to examine whether our G can be rewritten in an N=1 d=4 super-Poincare covariant notation using Green-Schwarz-like variables and, if this can be done, whether the obtained N=1 super-Poincare covariant action can be related to Berkovits' N=1 super-Poincare covariant action.

Our formulation is background independent only formally, because the star product was treated as an abstract object here, but the background dependence enters in the concrete expression of the star product.

The C-charge assignment on string fields causes the world-sheet ‘matter’ superconformal fields to carry C-charge. The background dependence of the BRST operator Q was eliminated by using the pure ghost operator Q_0 . Though the concrete form of the string fields is determined after the choice of the classical solution, the C-charge assignment will impose some restriction on the superconformal ‘matter’ fields dependence of the string fields. In this way, some background dependence may enter. This will be related to the fact that not all $N=1$ $c=15$ superconformal field theory backgrounds admit to construct an open RNS super-SFT action as was mentioned in [9].

In this paper, we considered $GSO(+)$ projected string fields only. So the theory contains BPS objects only. In order to be truly background independent, the pregeometrical theory must contain non-BPS objects as well as BPS objects. This implies that we need to extend the theory to include $GSO(-)$ string fields. Such an extension for Berkovits’ open NS super-SFT was given and shown to be very useful in the calculation of the tachyon potential in [16]. When Berkovits’ open RNS super-SFT’s are extended to include $GSO(-)$ projected string states, this theory will allow us to gain new insights into tachyon condensation, including the change of the number of the space-time supersymmetry generators. We believe that the corresponding pregeometrical formulation for this can be found along the line presented in this paper. Such a theory will be more fundamental background independent formulation, which describes non-BPS objects as well as BPS objects.

We used the operator Q_0 which has the non-trivial cohomology, so that the theory describes the physical open string excitations. Alternatively, if we choose a ghost c as Q_0 , then the theory is on the closed string vacuum, which is an end point of a tachyon condensation, because c has the trivial cohomology and so the theory does not describe any open string excitations. Recently, such a theory has attracted much attention as the vacuum SFT’s [15] which can be viewed as a pregeometrical formulation of the Witten’s bosonic SFT. We expect that our action with $Q_0 = c$ may provide an action for a vacuum open RNS super-SFT. It is interesting to examine this theory.

Covariant closed super-SFT without picture changing operators is not known at this moment. But some suggestions were given using the $N=4$ topological prescription [17, 14] in [8] and respecting the C-charge assignment in [9]. The pregeometrical formulation of closed super-SFT will provide more fundamental theory of them. It is interesting to formulate this theory in order

to gain some insights into the structures of closed super-SFT's.

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